

University of Milan - Bicocca

Ph.D. in Economics, Statistics and Data Science 2021 - 2022

Stochastic Processes

Introduction.

Definition of a stochastic process. Classification of general stochastic processes on the basis of their state space, index parameter and dependence relationships.

Markov chains.

Definitions. Examples: random walks. Higher-order transition probabilities. Existence of a Markov chain. Transience and persistence. Stationary distributions.

Martingales (discrete time)

Filtration. Definition of martingales, submartingales and supermartingales. Examples. Functions of martingales. Stopping times and optional sampling theorem. Inequalities. Convergence theorems.

Introduction to stochastic processes in continuous time

Filtration and stopping times. Martingales (continuous time). Markov property. Poisson process.

Brownian motion

Definition. Kolmogorov's existence theorem and application to Brownian motion. Martingale property. Quadratic variation. Markov property. Reflection principle, first passage time distribution and the distribution of the running maximum.

Stochastic calculus

Ito's integral: martingale property, Ito isometry and quadratic variation. Ito's formula for Brownian motion. Ito processes: definition and Ito's formula. Example: generalized geometric Brownian motion. Lévy's characterization theorem. Gaussian processes and Brownian bridge. Stochastic differential equations: Markov property and Feynman-Kac theorems.

Textbooks

Billingsley, P. *Probability and Measure* (3rd ed.).

Fristedt, B. and Gray, L. *A Modern Approach to Probability Theory*.

Shreve, S. E. *Stochastic Calculus for Finance II*.