

Empirical Bayes in Bayesian inference

A crucial aspect of Bayesian analysis is prior elicitation. In many prominent statistical problems the choice of prior hyperparameters highly affects inferential results, controlling e.g. model complexity in model selection frameworks, the degree of shrinkage or the sparsity level in regression or Gaussian sequence models. In all such cases, hyperparameter selection is quite delicate. From a genuinely Bayesian viewpoint, it should be based on the initial information set, available before collecting new data to be used for posterior computations. In the absence of sufficient prior information, a common practice is to opt for a data-dependent choice (empirical Bayes). A fully Bayesian alternative would consist in assigning a prior to hyperparameters, in a hierarchical Bayesian fashion. Three questions naturally arise:

1. What are principled ways to obtain a data-driven choice of a hyperparameter?
2. What are the main differences between inferences produced under the empirical and the hierarchical Bayesian approach, in practical, computational and theoretical terms?
3. Are there problems where a fully Bayesian formulation is hardly viable?

This reading group aims at reviewing the answers available so far in the literature and shading light on interesting directions for future research. A non-exhaustive list of relevant contributions follows.

Overviews and general methodologies

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Regression, sequence models and GAM's

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Non- and semiparametric problems

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